1. [Start of transcript. Skip to the end.](https://courses.edx.org/xblock/block-v1:ColumbiaX+CSMM.101x+3T2020+type@vertical+block@64f9c50cf25d4843841c4274fedcad2c?show_title=0&show_bookmark_button=0" \l "transcript-end-fcc754123be544e08d3c0357e4ab2bd4)
2. Let's start with the simplest truth table for negation.
3. If we have a proposition P, then the negation of P
4. is false if P is true, and the negation of P
5. is true if P is false.
6. So here you could see that we have a truth table of size 2.
7. That's 2 to the 1, the number of proposition you have.
8. So working on only one proposition,
9. negating it, so that the size is 2 to the 1.
10. And the function that we are building
11. is actually the function negating
12. P, which is equal to true or false, depending
13. on the value of the truth table of the proposition P.
14. How about conjunction?
15. Conjunction creates a new compound proposition
16. based on two propositions.
17. So given two propositions P and Q,
18. the compound composite proposition is P and Q,
19. and it would depend on the truth value of P and Q.
20. So the size of the truth table is
21. 2 to the 2, which is 4, because we
22. have two propositions that we are putting together
23. through the conjunction.
24. All right.
25. So the semantic of the conjunction is as follows.
26. If P is true and Q is true, then the compound is true.
27. If P is true and Q is false, then the compound is false.
28. If P is false and Q is true, then the compound is false.
29. If P is false and Q is false, then that compound is false.
30. In other words, the conjunction P and Q is true only
31. when we have both propositions are true at the same time.
32. If we think about, for example, the Wumpus World,
33. we're going to have Wumpus in 1, 2 and Wumpus in 2,1.
34. This is a false proposition because we
35. have only one Wumpus, and we can't have them
36. both in two cells.
37. So we must have one of them false so if any of them
38. is false, we are actually having all
39. of the outcome of the compound-- of the conjunction
40. is being false.
41. All right.
42. So now the disjunction.
43. Disjunction is similar in terms of truth table.
44. We also have here 2 to the 2 because we
45. have two propositions.
46. But for the disjunction, we are going
47. to have the disjunction weaker than a conjunction,
48. in the sense that whenever we have any of the proposition
49. true, we have the outcome to be true.
50. If both propositions are false, then we
51. have the outcome to be false.
52. If again, I take the proposition that Wumpus in 1, 2 or Wumpus
53. in 2, 1, it means that it's enough to have a Wumpus in one
54. of the two rooms to have actually
55. the proposition Wumpus in 1, 2 or Wumpus in 2, 1 to be true.
56. Exclusive or is another useful connector
57. that actually is true only when, and whenever there is only one
58. of the propositions true.
59. So as an example of this, the exclusive or,
60. you could think of if you go to the restaurant
61. and you have on the menu a soup or salad come in with the meal.
62. So you can't expect to have both of them.
63. So you expect to have at least one of them, right?
64. But you can't expect to have both of them.
65. So you have-- either you have a soup and not salad,
66. or you have a salad and not soup.
67. OK.
68. So basically this proposition here P exclusive or Q would be
69. true only if you have one of them-- exactly one of them--
70. equal to true.
71. So you can't have both of them.
72. And you can't you-- you are ensured
73. to have at least one entree coming in with the meal.
74. So again, here it's 2 to the 2.
75. That's going to be the truth table for the x or.
76. Here's another important truth table,
77. the truth table of implication.
78. Given proposition P and Q, P implies Q would be false only
79. when P is true and Q is false.
80. So this is a definition that logicians
81. decided to give to implication.
82. And this says that whenever the P is false,
83. we give the implication “truthfulness”,
84. the benefit of the doubt, so we going
85. to have false implies anything.
86. So the function would be true whenever--
87. if P is true, and Q is true, then the implication is true.
88. If P is false whenever Q is true or false,
89. the definition of P plus Q is true.
90. So this is the only case where the implication is actually
91. false.
92. Bi-directional or if and only if,
93. noted that as IFF, is a double implication.
94. So it's P implies Q and Q implies P. So we have P
95. and Q. P equivalent to Q is true only
96. when we have both P and Q true or both P and Q false.
97. So that's the main connectors in propositional logic.
98. Just like in arithmetic, there is a precedence of operators
99. for logical connectors.
100. For example, expressions in parenthesis
101. in propositional formulas are actually
102. processed from inside to outside.
103. We first give priority to the negation
104. if it is there, then the AND, then the OR, then
105. the implication, the bidirectional or if
106. and only if, otherwise we use left to right.
107. Whenever you have a doubt, just use
108. parenthesis to express your proposition
109. and avoid any confusion.
110. We can build larger and larger propositions.
111. For example, given the three propositions P, Q, and R,
112. it's possible to build the proposition that's
113. composed of--
114. using connectors composing P, Q and R. So for example,
115. P or Q implies R is a proposition.
116. This truth table is actually made of eight n-tuples
117. because we have 2 to the 3, given that we have
118. three propositions symbols.
119. Logical equivalence is another feature in propositional logic.
120. Two propositions P and Q are called
121. to be logically equivalent if and only
122. if the columns in the truth table are actually identical.
123. We write this as P equivalent, logically equivalent to Q or P
124. equivalent to Q. For example, if we take P and Q,
125. we know that actually the implication
126. P implies Q is logically equivalent to not P or Q.
127. You could see that if you do the truth table for these two
128. compound propositions, you are going
129. to get exactly the same truth value for the truth table.
130. So these two propositions are logically equivalent.
131. They have the same truth value for any compositions of P
132. and Q. So this is important.
133. It allows us to transform propositions
134. from one setting to another one.
135. Just like in arithmetic, we have some properties
136. on the operators, for example, the property of commutativity
137. holds for propositional logic symbols, for example.
138. P and Q is equal to Q and P. P or q is equal to Q or P.
139. But P implies Q is not equal to Q implies P. So be careful
140. with that.
141. Associativity.
142. If you have P and Q and R, you could just
143. change the order of the parenthesis.
144. Same thing for the disjunction.
145. The identity element is true.
146. For example, P and true is always P. P or true is true.
147. So we have since-- remember the definition of the disjunction.
148. Whenever we have one of the two elements
149. in the disjunction is true, we have a true for the compound
150. proposition.
151. Not not P is P. P and P is P. P or P
152. is P. We can distribute the conjunction over
153. to disjunction, and the disjunction
154. over the conjunction.
155. Also P and not P is false because one of them
156. must be false.
157. So the conjunction has to have-- requires that both of them
158. be true to be true.
159. P or not P is true because one of them
160. is guaranteed to be true.
161. We use an important feature called De Morgan's law
162. that allows us to introduce the negation on some compound,
163. conjunction, or disjunction.
164. The negation of a conjunction is the negation of the first term
165. or the negation of the second term.
166. The negation of a disjunction is a negation of the first term
167. and the negation of the second term.
168. We also like to talk about tautologies and fallacies
169. of contingencies.
170. A tautology is a proposition which is always true.
171. For example, suppose we have P. If I take P or not P,
172. P or not P .
173. So this is true.
174. We always have one of these two terms to be true.
175. Because the disjunction is always true.
176. So this is called a tautology.
177. We call contradiction or fallacy a proposition
178. that is always false.
179. Consider this one.
180. P and not P. I am upstairs and downstairs is something that
181. cannot be true at the same time.
182. So I'm going to have this false-- it's a fallacy--
183. for any value of P. Finally, a contingency
184. is a proposition which is neither
185. a tautology or a contradiction.
186. So it depends on different situations.
187. So we would have a mix of true and false in their truth value.
188. [End of transcript. Skip to the start.](https://courses.edx.org/xblock/block-v1:ColumbiaX+CSMM.101x+3T2020+type@vertical+block@64f9c50cf25d4843841c4274fedcad2c?show_title=0&show_bookmark_button=0#transcript-start-fcc754123be544e08d3c0357e4ab2bd4)